Putting it all together – Unifying frameworks.

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Unifying frameworks

- Many of the algorithms presented in the course are closely related.
- There have been attempts to investigate the theoretical relationship between the various methods.
- In this lecture, we will look at a theoretical framework that unifies many of the optimization methods that we have covered in this course. The framework is presented in the context of seeded image segmentation, but of course applies to other optimization problems as well.

Power waterheds

Camille Couprie, Leo Grady, Laurent Najman and Hugues Talbot
Power Watersheds: A Unifying Graph-Based Optimization Framework
IEEE Trans. on Pattern Analysis and Machine Intelligence, Vol. 33, No. 7, pp. 1384-1399, 2011.

Ali Kemal Sinop and Leo Grady *A Seeded Image Segmentation Framework Unifying Graph Cuts and Random Walker Which Yields A New Algorithm*
Proc. of ICCV, 2007

Recap, Random walker

Find a mapping $x : V \to [0,1]$ that minimizes

$$\left(\sum_{e_{ij} \in E} (w_{ij} |x_i - x_j|)^2 \right)^{1/2} , \tag{1}$$

subject to $x(F) = 1$ and $x(B) = 0$. A final segmentation s is given by

$$s_i = \begin{cases} 1 & \text{if } x_i \geq \frac{1}{2} \\ 0 & \text{if } x_i < \frac{1}{2} \end{cases} . \tag{2}$$

Recap, Random walker

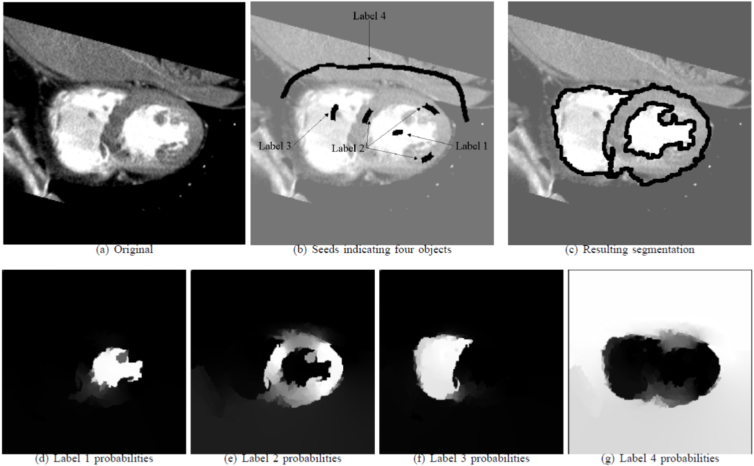


Figure 1: Seeded segmentation by random walker.

A general formulation of seeded segmentation

- In other words, the Random Walker method tries to minimize the l_2 norm of the difference in x between adjacent vertices.
- We have previously seen that the l_2 norm is a special case of a l_p norm.
- What happens if we try to extend the Random Walker method to other l_p norms?

A general formulation of seeded segmentation

Find a labeling $x : V \to [0, 1]$ that minimizes

$$\left(\sum_{e_{ij} \in E} (w_{ij}|x_i - x_j|)^q \right)^{1/q} , \qquad (3)$$

subject to $x(F) = 1$ and $x(B) = 0$. A final segmentation s is given by

$$s_i = \begin{cases} 1 & \text{if } x_i \geq \frac{1}{2} \\ 0 & \text{if } x_i < \frac{1}{2} \end{cases} . \qquad (4)$$

A general formulation of seeded segmentation

In the next few slides, we will see that the general formulation includes many of the algorithms we have convered in this course as special cases!

- For $p = 1$, we get the max flow/min cut problem.
- For $p = 2$, we get the Random walker problem. (By definition)
- For $p = \infty$, we get the shortest path problem.
- We will also extend the general formulation so that it includes minimum spanning forests/watersheds.

Case $q = 1$, Minimal graph cuts

If we substitute $q = 1$ into (3), we get

$$\sum_{e_{ij} \in E} w_{ij} |x_i - x_j| \ . \tag{5}$$

It was shown in [3, 2] that minimizing this equation subject to $x(F) = 1$ and $x(B) = 0$ is equivalent (dual) to the max flow problem. Thus, (6) can be minimized using, e.g., the Ford-Fulkerson algorithm described in lecture 4.

Case $q = \infty$, Shortest paths

If we let q approach ∞, we obtain the problem of minimzing

$$\max_{e_{ij} \in E} w_{ij} |x_i - x_j| \,, \tag{6}$$

subject to $x(F) = 1$ and $x(B) = 0$. It was shown in [3] that this is equivalent to segmentation by shortest path forests. Thus it can be solved by Dijkstra's algorithm.

Extending the framework to watersheds

To incorporate watersheds into the general framework, we separate the exponent on the weights from the exponent on the variables. We thus seek to minimize

$$\sum_{e_{ij} \in E} w_{ij}^{p} |x_i - x_j|^{q} \ . \tag{7}$$

subject to $x(F) = 1$ and $x(B) = 1$. When $p = q$, this is equivalent to the previous formulation (we can skip the root). When q is finite and $p \to \infty$, the results of the above optimization problem converges to MSF cuts (watersheds).

Unary terms

- So far, we have only considered binary terms ("interaction" between pairs of vertices).
- We can extend (7) further by including unary terms:

$$\sum_{e_{ij} \in E} w_{ij}^p |x_i - x_j|^q + \sum_{v_i \in V} w_{Fi}^p |x_i|^q + \sum_{v_i \in V} w_{Bi}^p |x_i - 1|^q \; . \tag{8}$$

- The unary terms can be incorporated by adding "phantom" seeds V_F and V_B.

Unary terms

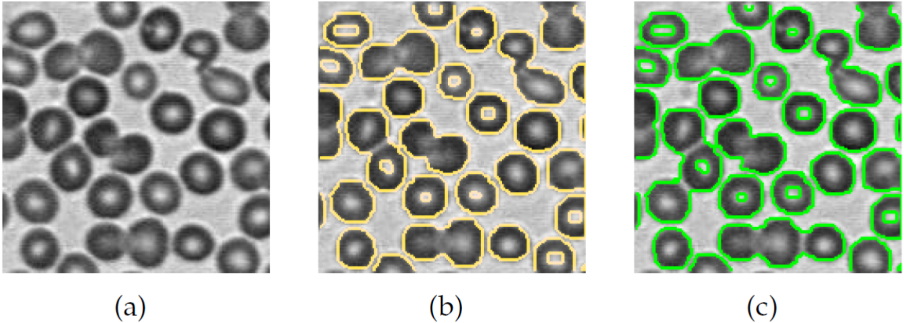
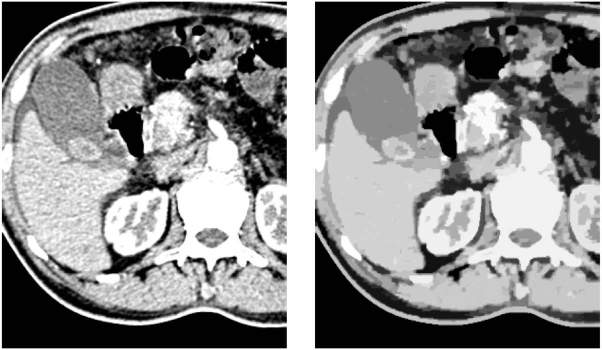


Figure 2: Unseeded segmentation with unary terms. (a) Image. (b) Segmentation by graph cuts. (c) Segmentation by watersheds.

Unary terms

In [1], Power watersheds with unary terms were used to compute anisotropic diffusion.



(a) Original image (b) PW result

Figure 3: Anisotropic diffusion with Power Watershed.

So, which method is better?

- Given the similarity between the presented method for seeded segmentation, how do we decide which one to use?
- In [2], an empirical comparison between a number of methods was presented.
- The study is based on the "Grabcut" database from Microsoft (available online). This dataset consists of 50 "natural" images provided with seeds and ground truth segmentations.

So, which method is better?

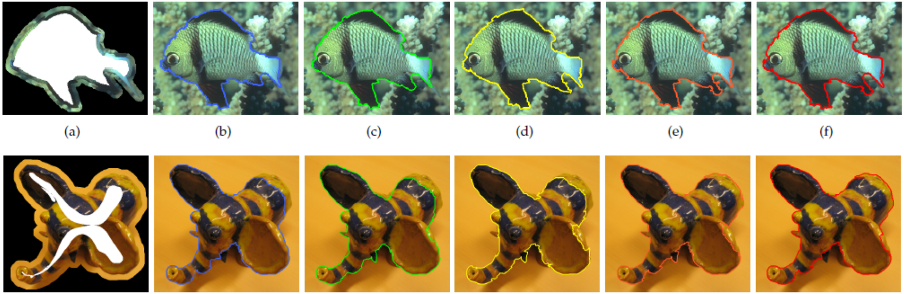


Figure 4: Example segmentations using the provided (top images) and skeletonized (bottom images) set of seeds on the Grabcut database images: (a) Seeds, (b) Graph cuts, (c) Random walker, (d) Shortest path, (e) Maximum spanning forest (standard watershed), and (f) Power watershed ($q = 2$).

Empirical comparison 1

	BE	RI	GCE	VoI	**Average rank**
Shortest paths	2.82	0.972	0.0233	0.204	**1**
Random walker	2.96	0.971	0.0234	0.204	**2.25**
MSF (Prim)	2.89	0.971	0.0244	0.209	**2.5**
Power wshed ($q = 2$)	2.87	0.971	0.0245	0.210	**3.25**
Graph cuts	3.12	0.970	0.0249	0.212	**5**

Figure 5: Results of comparison with symmetrically eroded seeds.

Empirical comparison 2

	BE	RI	GCE	VoI	**Average rank**
Graph cuts	4.70	0.953	0.0380	0.284	**1**
Power wshed ($q = 2$)	4.93	0.951	0.0407	0.297	**2.5**
Random walker	5.12	0.950	0.0398	0.294	**2.75**
MSF (Prim)	5.11	0.950	0.0408	0.298	**3.5**
Shortest paths	5.33	0.947	0.0426	0.308	**5**

Figure 6: Results of comparison with asymmetrically eroded seeds.

Computation time

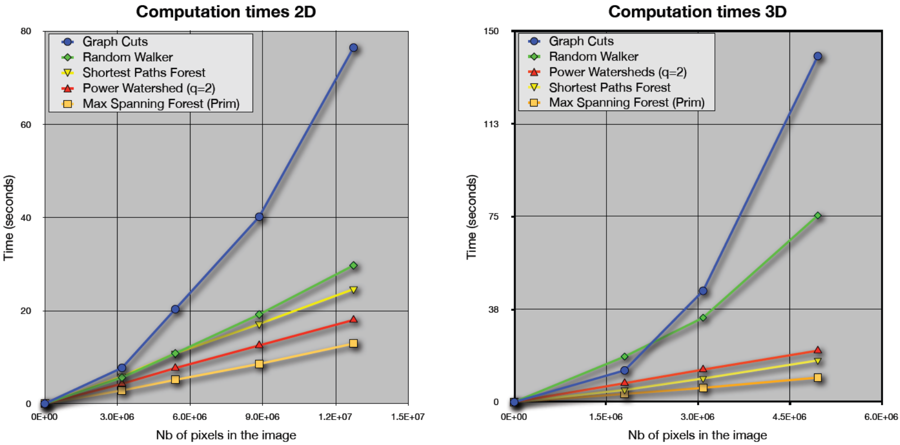


Figure 7: Computation time for the different algorithms in 2D and 3D.

Qualitative comparison

Min cut/max flow

+ Global optimization of weighted "area" (sum of edge weights in the cut).
+ Possible to approximate continuous "cut metrics" with arbitrary precision.
- Shrinking bias.
- Metrication artifacts on standard grids.
- NP-hard for more than two labels.
- Slower computation.

Qualitative comparison

Shortest paths

+ No shrinking bias.
+ Allows any number of labels.
+ Fast computation. Computation time indepent of the number of labels.
- Metrication artifacts on standard grids.
- Sensitive to noise and missing boundaries.

Qualitative comparison

MSF cuts

+ Global optimization of the max-norm of the cut.
+ Provably robust to variations in seed-point placement.
+ No shrinking bias.
+ Allows any number of labels.
+ Fast computation. Computation time indepent of the number of labels.
- Very sensitive to noise and leaks. (no penalty for "long" boundaries)

Qualitative comparison

Random walker

+ No shrinking bias.
+ Allows any number of labels.
+ No metrication artifacts.
+ Tolerant to noise and missing boundaries.
- Computation time depent of the number of labels.
- Slower computation.

Conclusions

- Many of the methods for seeded segmentation that we have seen in this course (RW, GC, MSF, SPF) can be formulated as minimizing the l_p norm of the gradients of a potential field with boundary conditions.
- The theoretical framework does not directly provide algorithms for optimizing the different cases, but it provides theoretical insight into the similarities and differences between the methods.
- The general optimization problem of seeded segmentation can be extended to include unary terms. This allows, e.g., the use of watersheds for general optimization in computer vision.
- We have looked at an empirical study that compares various methods for seeded segmentation.

References

[1] Camille Couprie, Leo Grady, Laurent Najman, and Hugues Talbot.
Anisotropic diffusion using power watersheds.
In *Image Processing (ICIP), 2010 17th IEEE International Conference on*, pages 4153–4156. IEEE, 2010.

[2] Camille Couprie, Leo Grady, Laurent Najman, and Hugues Talbot.
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[3] Ali Kemal Sinop and Leo Grady.
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In *Proceedings of the 11th International Conference on Computer Vision (ICCV)*. IEEE Computer Society, IEEE, 2007.